A solution to some right alternative ring problems

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Abstract. In this paper we show an automated construction of a counterexample to the TPTP problems RNG030 and RNG032. These problems have been open up to now. The used method shares some ideas with the Z-module method of [3], but it is more semantic oriented.
We also deal with ensuring program correctness. This is harder than in case of automated deduction, because there is no succinct witness of satisfiability.

1 Introduction

The TPTP is a library of test problems for evaluating theorem provers [5]. The TPTP is divided into domains. One of the TPTP domains is RNG, the domain containing ring theory problems. These problems are interesting because all the axioms are unit equational clauses (just equations) and there are associativity and commutativity axioms (AC axioms). Problems containing AC axioms are hard for general purpose theorem provers, so some problems remained open up to now. We tried to prove automatically the problems RNG30 and RNG32. They were open up to now. But even after we implemented quite sophisticated data structures for term indexing and different term orderings, the problems remained intractable.

Considering possibility that given conjectures can be false, we wrote a dedicated program that disproves these conjectures. If the conjecture is not a theorem then the set of the axioms does not imply the hypothesis, so the set of axioms and negated hypothesis is satisfiable. To prove satisfiability of the set of axioms and the negated hypothesis, one needs to show the existence of a structure that satisfies all the axioms and the negated hypothesis. Each theorem has a finite proof, but not every counterexample is finite (in such case a decision procedure would have existed). Our program constructs a counterexample that is finitely represented and can be easily transformed to a finite one. There are two interesting points about our method. The first point is that using some axioms properties (close relation to linear space) it is possible to get a succinct description of the counterexample. Basically, we will use the fact that a linear function is uniquely determined by the values on a basis. The second point is that used representation of a structure and the algorithm uses every derived equation to pruning search space.