Generalized Whac-a-Mole

Marcin Bienkowski[1], Marek Chrobak[2], Christoph Dürr[3], Mathilde Hurand[3], Artur Jeż[1], Łukasz Jeż[1], Jakub Łopuszański[1], and Grzegorz Stachowiak[1]

[1] Institute of Computer Science, University of Wrocław, 50-383 Wrocław, Poland.
[2] Department of Computer Science, University of California, Riverside, CA 92521, USA.
[3] CNRS, LIX UMR 7161, Ecole Polytechnique 91128 Palaiseau, France.

Abstract. We consider online competitive algorithms for the problem of collecting weighted items from a dynamic set $\mathcal{S}$, when items are added to or deleted from $\mathcal{S}$ over time. The objective is to maximize the total weight of collected items. We study the general version, as well as variants with various restrictions, including the following: the *uniform case*, when all items have the same weight, the *decremental sets*, when all items are present at the beginning and only deletion operations are allowed, and *dynamic queues*, where the dynamic set is ordered and only its prefixes can be deleted (with no restriction on insertions). The dynamic queue case is a generalization of bounded-delay packet scheduling (also referred to as buffer management).

We present several upper and lower bounds on the competitive ratio for this problem, including the following. For the general version, a simple greedy algorithm is 2-competitive and no better ratio is possible for deterministic algorithms. This lower bound extends, in fact, to randomized algorithms against an adaptive adversary. For the uniform case and decremental sets we give an $\mathrm{e}/(\mathrm{e}-1)$-competitive randomized algorithm (against an oblivious adversary) and prove a matching lower bound.

Most of our results concern dynamic queues. For decremental queues, we present a 1.737-competitive deterministic algorithm and prove a lower bound of ≈ 1.63. For the FIFO case, when insertions are allowed only at the end (this generalizes scheduling packets with agreeable deadlines), we give an algorithm with ratio 1.8. For general dynamic queues, we give a deterministic algorithm that has competitive ratio ϕ for instances with non-decreasing weights. In the randomized case, we show that no memoryless algorithm can have ratio smaller than $\mathrm{e}/(\mathrm{e}-1)$ against an adaptive adversary, matching a known upper bound.